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Collective Effects From Induced Behaviour

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Abstract. We present a solvable model for describing quantitatively situations where the individual behaviour of agents in a group "percolates" to collective behaviour of the group as a whole as a result of mutual influence between the agents.

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1 Introduction

One very common interaction between the individuals of a group is the tendency to imitate each other. Under certain conditions this might be expected to critically affect the behaviour of the group as a whole. It may be interesting to have a quantitative understanding of the conditions under which individual patterns of behaviour may propagate to a group behaviour as a result of mutual influence among a few neighbors. As another example we may consider the problem of ensuring the reliability in functioning of a complex machinery by increasing redundancies. Typically the "parallel" components cannot be made completely independent. How does reliability generally depend on redundancy if failure propagation is possible? This is one major question in security analyses [1]. There are further examples with similar problem setting: the dynamics of certain phase transitions, critical reactions, epidemic models, etc.

Here we would like to analyze how *induced behaviour* can lead to *collective effects* in the frame of a probabilistic model introduced earlier [2] and which can be solved exactly. Thereby we restrict ourselves to the very elementary mechanism of imitation and we do not attempt to include more refined interactions, such as beliefs, goals, cooperation, competition, etc – see, e.g., [3]. For this case we can provide a solution in closed form. The model will be described and discussed in section 2 and its solution in section 3. Here we also define a Monte-Carlo simulation by interpreting the closed solution as a partition function. Approximations permiting some qualitative insight are discussed in Section 4. In Section 5 we present and discuss the results for some representative cases. Here we use the Monte-Carlo simulation to treat large ensembles of agents with "realistic" correlations.

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2 The probabilistic model

We consider a set $\mathcal{N}$ consisting of n points, labeled $i = 1, \ldots, n$, each of which can *spontaneously* burst (and "disappear") with probability w_{0i} and let K_{ij} be the *induced* probability that point i bursts *because* j *has* bursted.

If these points represent the parallel components of a machinery, the functioning of the latter is ensured as long as at least one of the components works. For the behaviour of a group of agents the relevant question is again whether (practically) all agents show the same behaviour. The key quantity is therefore the probability with which all points have bursted:

$$W = W(n) = W(n; \{w_0, K\}). \tag{1}$$

We also define the "no-propagation" probabilities

$$L_{ij} = 1 - K_{ij} \tag{2}$$

and introduce the following simplifying assumptions:

1) Symmetry:
$$K_{ij} = K_{ji}; \tag{3}$$

2) Independence of "no-propagation" events:
$$L_{1(23)} = L_{12} L_{13}, \tag{4}$$
i.e. the probability that point 1 bursts because 2 and 3 have both bursted is
$$K_{1(23)} = 1 - (1 - K_{12})(1 - K_{13}) = K_{12} + K_{13} - K_{12} K_{13}. \tag{5}$$

Notice that the absence of time evolution in this model implies that, in a real situation, the whole development is expected to take place in a very short time such that no parameter changes appreciably. The simplifying assumptions 1), 2) are of course limitations, we can imagine, however, many realistic situations under which they hold approximately. Symmetry, for instance, may well be expected to hold on the average in a group of similar agents (the birds in a flock, say). The independence assumption depends on the real interactions.

For illustration consider the following "temperature model": 3 points isolated in an enclosure, with the bursting probability for point '1' described by some monotonic function of the ambient temperature. Assume that bursting of a point increases the average temperature by ΔT. We ignite points '2' and '3' and see what happens with '1'. If only '2' *or* '3' had bursted, '1' will go off with probability $P(\Delta T)$, while if both '2' *and* '3' went off the ambient temperature is $2\Delta T$ and '1' will explode with probability $P(2\Delta T)$. Then (5) would require:

$$P(2\Delta T) = 2P(\Delta T) - P(\Delta T)^2, \tag{6}$$

or, with $h(T) = -ln(1 - P(T))$,

$$h(2T) = 2h(T), \tag{7}$$

with solution $h(T) = aT$. Hence in this "temperature model" only:

$$P(T) = 1 - \mathrm{e}^{-aT} \tag{8}$$

is compatible with (5). In particular, a threshold behaviour like $P(T) = \theta(T - T_0)$ will violate (6) if $\Delta T < T_0 < 2\Delta T$. We shall retain, however, for this analysis the independence assumption (we shall indicate below how one can relax it when the dynamics of the interaction is known).

Before we proceed and solve the model notice the following four limiting cases:

$$K_{ij} = 0: \quad W = W_0(n) = \prod_{i=1}^{n} w_{0i}; \tag{9}$$

$$K_{ij} = 1: \quad W = W_1(n) = 1 - \prod_{i=1}^{n}(1 - w_{0i}); \tag{10}$$

$$w_{0i} = 0: \quad W = 0; \tag{11}$$

$$w_{0i} = 1: \quad W = 1. \tag{12}$$

They are helpful for tests, normalization, etc.

3 Exact solutions

3.1 Combinatorial solution.

Starting to solve the model we first remark:

in following the propagation of the bursts each K_{ij} can be used only once: there is no way to get something like K_{ij}^2.

It is convenient to introduce the notation $\alpha = (ij)$ for the non-ordered pair $\{i, j\}$. There are $\frac{1}{2}n(n-1)$ different α's, $\{\alpha\} = \Omega$. Let ω denote a subset of α's; there are $2^{\frac{1}{2}n(n-1)}$ different ω's (including the empty set $\emptyset$). For the time being we shall take $w_{0i} = w_0$ independent on i, then W is of the form:

$$W = W(n, w_0, \{K_\alpha\}) = \sum_\omega C_\omega \prod_{\alpha\in\omega} K_\alpha, \tag{13}$$

with the last factor taken to be 1 for $\omega = \emptyset$. Putting $K = 1$ on a subset of Ω and 0 in the rest we define:

$$W_\omega = W|_{K_\alpha=1 \text{ if } \alpha\in\omega,\ K_\alpha=0 \text{ if } \alpha\notin\omega} \tag{14}$$

and we have:

$$W_\omega = \sum_{\omega'\subseteq\omega} C_{\omega'} \prod_{\alpha\in\omega'} K_\alpha|_{K_\alpha=1} = \sum_{\omega'\subseteq\omega} C_{\omega'}. \tag{15}$$

Let for $\omega' \subset \omega$:

$$\pm_{\omega\omega'} = (-1)^{(\text{nr. of elements of } \omega)-(\text{nr. of elements of } \omega')}, \tag{16}$$

then we can invert (15) to obtain

$$C_\omega = \sum_{\omega'\subseteq\omega} \pm_{\omega\omega'} W_{\omega'}. \tag{17}$$

We next evaluate W_ω. Each ω achieves a partition of $\mathcal{N}$ in the following way: if $\{i, j\} \in \omega$ we join the points i and j. Thus $\mathcal{N}$ is partitioned into (non-empty) connected subsets $\nu_k^{(\omega)}$ labeled with the index k and containing $n_1^{(\omega)}, n_2^{(\omega)}, \ldots$ points, $n_k^{(\omega)} > 0$, such that $\sum_k n_k^{(\omega)} = n$. Then:

$$W_\omega = \prod_k \left(1 - (1 - w_0)^{n_k^{(\omega)}}\right) \tag{18}$$

and thus from (17), (13)

$$W = \sum_{\omega\subseteq\Omega} \sum_{\omega'\subseteq\omega} \pm_{\omega\omega'} \prod_k \left(1 - (1 - w_0)^{n_k^{(\omega')}}\right) \prod_{\alpha\in\omega} K_\alpha. \tag{19}$$

After rearranging the terms using (16), (19) gives:

$$W(n; w_0, \{K\}) = \sum_{\omega\subseteq\Omega} \prod_k \left(1 - (1 - w_0)^{n_k^{(\omega)}}\right) \times \prod_{\alpha\in\omega} K_\alpha \prod_{\alpha'\in C_\omega^\Omega} (1 - K_{\alpha'}). \tag{20}$$

The extension to different w_{0i} is straightforward and leads to the general solution:

$$W(n; \{w_0, K\}) = \sum_{\omega\subseteq\Omega} \prod_k \left(1 - \prod_{i\in\nu_k^{(\omega)}} (1 - w_{0i})\right) \times \prod_{\alpha\in\omega} K_\alpha \prod_{\alpha'\in C_\omega^\Omega} (1 - K_{\alpha'}). \tag{21}$$

The sum over ω is taken over all subsets of Ω, including the empty set and Ω, where Ω is the set of all α's, and C_ω^Ω is the complement of the set ω in Ω.

3.2 Iterative solution.

It is helpful to write down also an iterative solution of the model. Consider a set η of points out of which only a subset σ is still untouched, and consider all the ways the failure can propagate from the points in C_σ^η to those in σ, then the probability that also σ blows up is:

$$P(\eta|\sigma) = \sum_{\tau\subset\sigma}\prod_{i\in C_\tau^\sigma}\left(1-\prod_{j\in C_\sigma^\eta}L_{ij}\right)\times \prod_{i\in\tau}\left(\prod_{j\in C_\sigma^\eta}L_{ij}\right)P(\sigma|\tau),\quad P(\eta|\emptyset)=1 \tag{22}$$

and we have:

$$W(n;\{w_0,K\}) = \sum_{\sigma\subset\mathcal{N}}\prod_{i\in C_\sigma^{\mathcal{N}}}w_{0i}\prod_{j\in\sigma}(1-w_{0j})P(\mathcal{N}|\sigma). \tag{23}$$

In (22,23) all inclusions are strict and go also over the empty set.

For the case of "homogeneous interaction"

$$K_{ij}=K,\quad L_{ij}=L, \tag{24}$$

(22,23) simplify considerably:

$$P_{m,l} = \sum_{k=0}^{l-1}\binom{l}{k}(1-L^{m-l})^{l-k}(L^{m-l})^l P_{l,k} \tag{25}$$

$$W(n;w_0,K) = \sum_{m=0}^{n-1}\binom{n}{m}w_0^{n-m}(1-w_0)^m P_{n,m}. \tag{26}$$

Finally let us remark that in the frame of an explicit model for the interaction one can relax the assumption (5) and directly construct the compound probabilities appearing in (22). For instance, for the "temperature model" of section 2 we only need to substitute in (22):

$$\prod_{j\in C_\sigma^\eta}L_{ij}\to 1-P_i\left(\sum_{j\in C_\sigma^\eta}T_{ij}\right), \tag{27}$$

where T_{ij} is the increase in temperature at site 'i' due to the bursting of point 'j' and $P_i(T)$ is the probability that point 'i' explodes when the ambient temperature is T.

3.3 Monte Carlo analysis

A Monte Carlo simulation can be set up based on (20,21). We define for arbitrary p partition functions:

$$\mathcal{Z}_p = \sum_{\omega\subseteq\Omega}\mathcal{W}_0(\omega)^p\prod_{\alpha\in\omega}K_\alpha\prod_{\alpha'\in C_\omega^\Omega}(1-K_{\alpha'}), \tag{28}$$

where

$$\mathcal{W}_0(\omega)=\prod_k\left(1-(1-w_0)^{n_k^{(\omega)}}\right) \tag{29}$$

and we have (see (12)):

$$\mathcal{Z}_0=1,\quad \mathcal{Z}_1=W(n). \tag{30}$$

Starting from any partition function $\mathcal{Z}_p$ we can write $W(n)$ as an average:

$$W(n)=\frac{\langle\mathcal{W}_0^{1-p}\rangle_p}{\langle\mathcal{W}_0^{-p}\rangle_p}, \tag{31}$$

$$\langle\mathcal{W}_0^q\rangle_p=\frac{1}{\mathcal{Z}_p}\sum_{\omega\subseteq\Omega}\mathcal{W}_0(\omega)^q\mathcal{W}_0(\omega)^p\prod_{\alpha\in\omega}K_\alpha\prod_{\alpha'\in C_\omega^\Omega}(1-K_{\alpha'}),$$

in particular (see (30)):

$$W(n)=\langle\mathcal{W}_0\rangle_0. \tag{32}$$

The MC procedure uses the terms in $\mathcal{Z}_p$ as Boltzmann-Gibbs factors to achieve an importance sampling of ν_k^ω partitions. In actual simulations using $p=0$, 0.5 or 1 the results were similar, therefore we used for the systematic analysis $p=0$, i.e. eq. (32), which is faster. Then the Metropolis algorithm, which produces new partitions by adding or removing "bonds" $\alpha=(ij)$ one at a time, is local (and vectorizable). Note that since the $\mathcal{W}_0$ are positive, lack of convergence in the MC simulation based on (32) is likely to show up as underestimation of the exact result.

Whenever we could compare the results of the Monte Carlo simulation with exact summation of either the combinatorial (20,21) or the iterative (22,23) solution we have found very good agreement – see section 5.

4 Approximations

4.1 Mean Field approximation

We introduce an "effective" bursting probability w_i via the consistency equation

$$1-w_i=(1-w_{0i})\prod_{j\neq i}(1-w_jK_{ij}). \tag{33}$$

We shall in the following assume translational invariance, that is, $w_{0i}=w_0$ and $K_{ij}=K_{|i-j|}$. For finite systems we shall assume periodic boundary conditions. Then the mean field equation reads:

$$1-w=(1-w_0)\exp\left(\sum_{\nu=1}^{n-1}\ln(1-wK_\nu)\right). \tag{34}$$

w can be found iteratively. A rough estimate is:

$$w\simeq 1-(1-w_0)\mathrm{e}^{-\lambda\mathcal{K}},\quad \mathcal{K}=\sum_{\nu=1}^{n-1}K_\nu \tag{35}$$

with some $\lambda\sim\mathcal{O}(1)$. The parameter $\mathcal{K}$ has an intuitive meaning: it gives the average number of points which can be affected by one point. We then have:

$$\ln W(n)=n\ln w\sim-(1-w_0)\mathrm{e}^{\ln n-\lambda\mathcal{K}}, \tag{36}$$

which indicates that the behaviour of $W(n)$ is determined by the dependence on n of $\mathcal{K}$.

4.2 A lower limit

A more refined approximation can be derived which, for the case of homogeneous interaction (24) provides a lower bound for $W(n)$. We start from the iterative solution (22,23) and assume that for some λ we have:

$$P(\sigma|\tau) \geq \prod_{i\in\tau}(1-F_i(\sigma;\lambda)) \tag{37}$$

$$F_i(\sigma;\lambda) \equiv \prod_{j\in\sigma,\ j\neq i}(1-\lambda K_{ij}) \tag{38}$$

(here $\emptyset \subset \tau \subset \sigma \subset \eta$). Since all contributions to (22) are positive we can then write:

$$\begin{aligned} P(\eta|\sigma) &\geq \sum_{\tau\subset\sigma}\left[\prod_{i\in C_\tau^\sigma}\left(1-\prod_{j\in C_\sigma^\eta}L_{ij}\right)\right]\times \\ &\left[\prod_{i\in\tau}\left(\prod_{j\in C_\sigma^\eta}L_{ij}\right)\right]\left[\prod_{i\in\tau}(1-F_i(\sigma;\lambda))\right] \\ &= \prod_{i\in\sigma}\left(1-F_i(\sigma;\lambda)\prod_{j\in C_\sigma^\eta}L_{ij}\right)- \\ &\prod_{i\in\sigma}\left((1-F_i(\sigma;\lambda))\prod_{j\in C_\sigma^\eta}L_{ij}\right) \geq \prod_{i\in\sigma}(1-F_i(\eta;\lambda)) \end{aligned} \tag{39}$$

If we can prove that there exists a $0 \leq \lambda \leq 1$ such that the last inequality holds, then we have:

$$\begin{aligned} W(n;\{w_0,K\}) \geq &\prod_{i\in\mathcal{N}}(1-F_i(\mathcal{N};\lambda)(1-w_{0i}))- \\ &\prod_{i\in\mathcal{N}}(1-F_i(\mathcal{N};\lambda))(1-w_{0i}). \end{aligned} \tag{40}$$

In the homogeneous interaction case we can show that a $\lambda > 0$ can always be found such that (39) holds. We have:

$$F_i(m;\lambda) = (1-\lambda K)^{m-1} \tag{41}$$

and we must show that for all $m,\ l < m$:

$$\begin{aligned} &\left[1-(1-\lambda K)^{l-1}(1-K)^{m-l}\right]^l \geq \left[1-(1-\lambda K)^{m-1}\right]^l \\ &+\left[(1-(1-\lambda K)^{l-1})(1-K)^{m-l}\right]^l. \end{aligned} \tag{42}$$

The expression in the square brackets on the left hand side decreases with increasing l, while those on the right hand side do not. Therefore the worst case is $l = m-1$ and it is enough to prove that:

$$\begin{aligned} &\left[1-(1-\lambda K)^{m-2}(1-K)\right]^{m-1} \geq \left[1-(1-\lambda K)^{m-1}\right]^{m-1} \\ &+\left[(1-(1-\lambda K)^{m-2})(1-K)\right]^{m-1}. \end{aligned} \tag{43}$$

For $m = 2$ (43) is satisfied for any $\lambda < 1$. For $m \sim 1/K$ (43) is satisfied for $\lambda \geq 1/2$ and with increasing m the bound on λ goes toward 1. More precise numerical bounds are given in Fig. 1. For the general correlation case we may use (40) as an alternative to the mean field approximation.

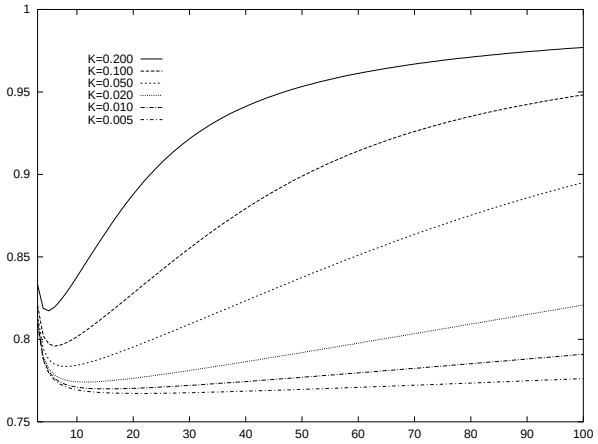

Fig. 1. *λ satisfying (43) vs n for various K. For values of λ below the curves the formulae (38,40) provide lower bounds.*

5 Results and Discussion

In the introduction we asked about the collective effects of induced behaviour of agents in an ensemble. As we have noticed, various specific questions can be asked in this context. They can all be subsumed under some general questions, which in the above model can be exemplified as follows:

Question 1: *How does the total burst (failure) probability $W(n, w_0, \{K_{ij}\})$ behave with increasing n for various types of "aggregation", distinguished by the way in which the mutual influence between systems depends on n?*

As an instructive example we consider a 1-dimensional ensemble and put n points equidistantly on a circle. We assume "finite correlation length" ξ:

$$K_{ij} = a\mathrm{e}^{-d_{ij}/\xi}, \tag{44}$$

$$d_{ij} = \min(|i-j|, n-|i-j|), \tag{45}$$

with

$$\xi(n) = \xi_0 n^\alpha. \tag{46}$$

Hence for $\alpha = 1$ we have an *intensive* aggregation (more and more points come under the influence of a single one while the size of the ensemble measured in correlation lengths stays fixed) and for $\alpha = 0$ an *extensive* aggregation (the density of points stays constant while the total volume increases). $0 < \alpha < 1$ interpolates between these situations (we are not concerned here with the dynamics of the aggregation: attraction, repulsion etc but just take the aggregation law as given).

A rough impression is offered by the mean field approximation. For large n and small w_0, K:

$$\mathcal{K} \simeq a\xi_0 n^\alpha \tag{47}$$

and

$$\ln W(n) \sim -(1-w_0)\exp\left(\ln n - a\lambda\xi_0 n^\alpha\right), \tag{48}$$

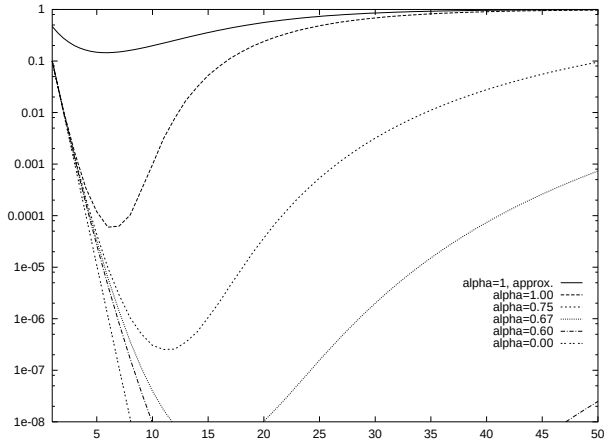

Fig. 2. *$W(n)$ vs n from the mean field approximation using the iterative solution of eq. (34); the line identified by "approx." uses the further approximation (48). We use $w_0 = 0.1,\ a = 0.3,\ \xi_0 = 1/3.5$ – see (44-46). The different aggregation types are identified by α.*

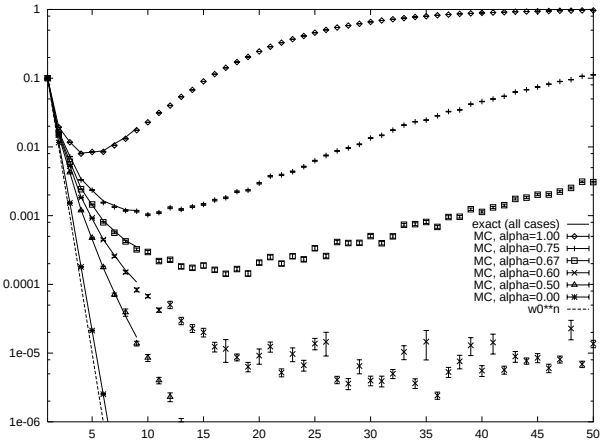

Fig. 4. *$W(n)$ vs n from the exact summation (lines) and Monte Carlo simulation (points with error bars) for various aggregation types, for $w_0 = 0.1,\ a = 0.3,\ \xi_0 = 1/3.5$. Also shown is w_0^n, for comparison.*

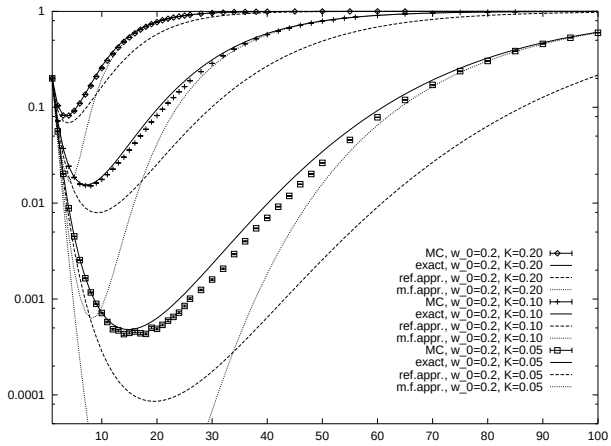

Fig. 3. *$W(n)$ for homogeneous interaction (infinite correlation length) from: the Monte Carlo simulation, the exact summation of the iterative solution (25,26), the "refined approximation" (40) (with the corresponding minimal values of λ from Fig. 1) and the mean field approximation (34). Here $w_0 = 0.2,\ K =$ 0.05, 0.1 and 0.2.*

see (34)-(36), which has a minimum for:

$$n_{\min} \sim (a\alpha\lambda\xi_0)^{-\frac{1}{\alpha}}\,, \tag{49}$$

above which $W(n)$ goes to 1 for all $\alpha > 0$. In Fig. 2 we illustrate this behaviour, both from formula (48) and using an iterative solution of eq. (34).

Hence it appears that a drastic change in the group behaviour is expected to occur when the average degree of mutual influence represented by (35,47) starts to compensate for the statistical "insignificance" of an individual (observe the compensation in the exponent of eq. (48)).

We now turn to the exact solutions to see more precisely what happens. Since the number of partitions in the sum in (21) increases as $2^{\frac{n(n-1)}{2}}$ we could sum exactly the combinatorial formula (21) to evaluate $W(n)$ only for n up to ~ 7. An exact algorithm based on the iterative solution (22,23) allowed us to go up to n about 11, we restricted ourselves to $n = 9$ for systematic runs. Large n were reached by Monte Carlo analysis using (28-32).

For a check of the MC accuracy we compare in Fig. 3 the MC calculation with the exact summation for the case of homogeneous interaction (25,26) (infinite correlation length ξ). Here and below the errors quoted do not account for statistic correlations in the data and are only indicative for the stability of the latter. In the region of very small probabilities the Monte Carlo data for large n are seen to systematically underestimate the result by up to 30%, which may be due to insufficient thermalization of our runs (we start with a random bond configuration and perform 10000 thermalization sweeps at each n) – otherwise the agreement is very good.

We also can see on Fig. 3 that the "refined approximation" of section 4.2 does provide a lower limit and is better than the mean field approximation in the region of the minimum of $W(n)$, while the latter describes more accurately the asymptotic regime. Both of them, however, are rather far from the exact and MC results. While providing qualitative insights and predicting correctly the *position* of the minimum and the asymptotic behaviour, the mean field approximation fails even at the semi-quantitative level: the value of $W(n)$ near the minimum is underestimated by orders of magnitude.

For the more realistic (finite correlation length ξ) case we show in Fig. 4 numerical results (exact summation and Monte Carlo simulation) for distance dependent interaction K (44,45), using $w_0 = 0.1,\ a = 0.3$ and $\xi_0 = 1/3.5$ for various types of aggregation: $\alpha = 0,\ 0.5,\ 0.6,\ 0.65,\ 0.75$ and 1 (46). We see that even for small α (extensive aggre-

gation) the presence of correlations can increase $W(n)$ by a large factor. The most interesting result is, however, the indication of the existence of an α_0 much below 1, such that for $\alpha > \alpha_0$ $W(n)$ does indeed develop a minimum after which it grows to 1 as suggested by (49) and (48). The minimum is rather shallow and can appear already at small n. Hence, ensembles which do not "expand" fast enough with increasing number of points (i.e., for which the size of the system measured in correlation lengths, n/ξ increases only as a small power of n, $1-\alpha < 1-\alpha_0$) are intrinsically unstable under induced behaviour.

As a side remark, we notice again the large difference to the mean field approximation, especially in the interesting turn over region – compare Figs. 2 and 4. This points to the benefit of having exact solutions and algorithms allowing faithful numerical analyses.

Consider now an ensemble whose spatial organization can vary, for fixed n, then we can ask:

Question 2: *Assuming a constant interaction scale, how does W behave if the density of the points varies?*

Roughly, this means that the strength of the correlation varies. In Fig. 5 we show the dependence of W on the density ρ (i.e., on K) for the homogeneous interaction case, $\xi \to \infty$ (25,26) (exact summation) for $n = 25,\ 50, 100$ and 200 using the *ad hoc* rule:

$$K(n) = \rho(20/n)^{0.78} \tag{50}$$

to bundle the data and allow comparison of various n.

For the general case (finite ξ) we show in Fig. 6 W from the Monte Carlo simulation for three values of n as function of the density ρ, where we take

$$\xi(n, \rho) = \xi_0 \rho. \tag{51}$$

We see that fluctuations of the density can easily destabilize the ensemble if the latter is near some "critical" density, $\rho_c(n)$. For large n the critical fluctuations appear to be $\propto const/\sqrt{n}$.

Also other ways of introducing a scale or for posing the stability question can be imagined. In the above discussion the parameters have been chosen more or less arbitrarily. Of course, the explicit results depend on the particular problem: the form of the function $K(d)$, the spatial arrangement, the aggregation form etc. It seems, however, that we see here a generic feature of induced behaviour, namely the capability to produce collective effects and that we are able in the frame of this probabilistic model to provide a quantitative analysis of this capability.

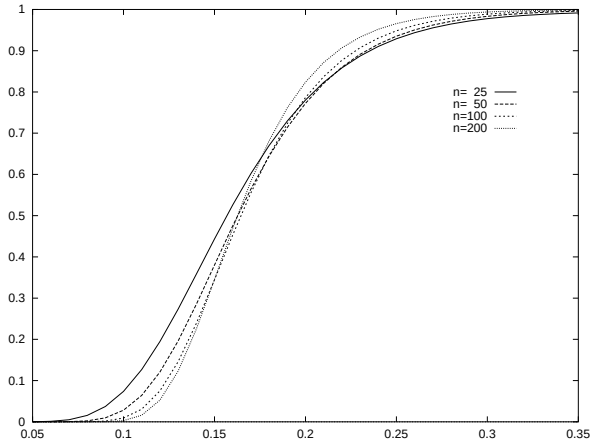

Fig. 5. *The dependence of W on K for the homogeneous interaction case (infinite correlation length) (25,26) for $w_0 = 0.2$ and $n = 25,\ 50,\ 100$ and 200. W is plotted here vs ρ (the density) using formula (50) to bundle the curves.*

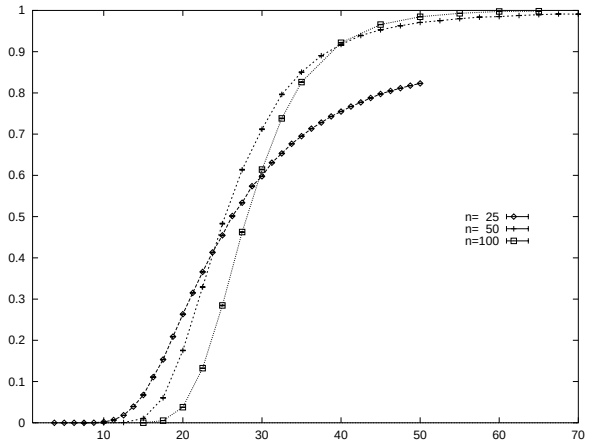

Fig. 6. *W vs ρ (the density) for the general case from the Monte Carlo simulation for $n = 25$, $n = 50$ and $n = 100$ and $w_0 = 0.1$, $a = 0.3$ and $\xi_0 = 1/3.5$.*

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